Family of curves

Cartesian coordinates

In an x-y Cartesian coordinate system, the most general form of equation of a siluroid is:

[1]

$$\left(x^2+y^2\right)^2-4xn\left(x^2-y^2\right)=0$$

where n is a natural integer, that is $n \in \mathbb{N}$.

Polar coordinates

Analogously, the most general form of equation of a siluroid in Polar coordinates is:

[2]

$$\rho = 4n\cos(\theta)\cos(2\theta)$$

Parametric formulas

The parametric equations of a siluroid are:

[1a]

$$\begin{cases} x = \dfrac{2n + t}{2n} t \\ y = \pm \dfrac{\sqrt{4n^2 - t^2}}{2n} t \end{cases} \qquad \textit{where } t \in [-2n,\ 2n]$$

Mother curve

If $n = 1$ we obtain the so called *mother curve*. Note that the mother curve is a particular case of a more general curve called *folium*. In particular it is a *trifolium* where $a = {}^{1}/_{2}$

Cartesian coordinates

In Cartesian coordinates, the mother curve is:

[3]

$$\left(x^2+y^2\right)^2-4x\left(x^2-y^2\right)=0$$

Polar coordinates

In Polar coordinates, the mother curve is:

[4]

$$\rho = 4\cos(\theta)\cos(2\theta)$$

Solutions

The solutions for the variable y are:

[5]

$$y = \pm\sqrt{-x^2 \pm 2\sqrt{x^2\left(2x+1\right)} - 2x}$$

Parametric formulas

The parametric equations of the mother curve are:

[3a]

$$\begin{cases} x = \dfrac{2 + t}{2}\, t \\ y = \pm\dfrac{\sqrt{4-t^2}}{2}\, t \end{cases} \qquad where\; t \in [-2,\ 2]$$

Calculus

Integrals

Since the area of a curve in polar coordinates $\rho(\theta)$ between the angles α and β is

[11]

$$A = \frac{1}{2}\int_{\alpha}^{\beta} \rho^2 \,\mathrm{d}\,\theta$$

it is possible to demonstrate that the area of the siluroid between two generic angles is

[12]

$$A = n^2\left(2\theta + \sin\left(4\theta\right) + 2\sin\left(2\theta\right) - \frac{2}{3}\sin^3\left(2\theta\right)\right)\Bigg|_{\alpha}^{\beta}$$

Total area

The total area of the siluroid is $2\pi n^2$

Main lobe

The area of the main lobe is

[13]

$$\left(\pi + \frac{8}{3}\right)n^2$$

Secondary lobe

The area of a secondary lobe is

[14]

$$\frac{1}{2}\left(\pi - \frac{8}{3}\right)n^2$$

Derivatives

Let us use the equation in polar coordinates to calculate the derivatives of a siluroid. The first derivative is

[15]

$$\dot{\rho} = -4n\sin(\theta)(3\cos(2\theta) + 2)$$

whereas the second derivative is

[16]

$$\ddot{\rho} = -4n\cos(\theta)(9\cos(2\theta) - 4)$$